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A Solution of the Randall-Sundrum Model and the Mass Hierarchy Problem

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Abstract

A solution of the Randall-Sundrum model for a simplified case (one wall) is obtained. It is given by the $1/k^2$-expansion (thin wall expansion) where $1/k$ is the *thickness* of the domain wall. The vacuum setting is done by the 5D Higgs potential and the solution is for a *family* of the Higgs parameters. The mass hierarchy problem is examined. Some physical quantities in 4D world such as the Planck mass, the cosmological constant, and fermion masses are focussed. Similarity to the domain wall regularization used in the chiral fermion problem is explained. We examine the possibility that the 4D massless chiral fermion bound to the domain wall in the 5D world can be regarded as the real 4D fermions such as neutrinos, quarks and other leptons.

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1 Introduction

In nature there exists the mass hierarchy such as the Planck mass (10^{19}GeV), the GUT scale (10^{15}Gev), the electro-weak scale (10^2GeV), the neutrino mass($10^{-11} - 10^{-9}$GeV) and the cosmological size(10^{-41}GeV). How to naturally explain these different scales ranging over 10^{60} (so huge !) order has been the long-standing problem (the mass hierarchy problem). One famous approach is the Dirac's large number theory[1]. He tried to explain some ratios between basic physical quantities (the electric force/the gravitational force, the age of the universe/the period during the light's passing through the (classical) electron, the total mass in the universe/the proton mass) using the idea of the variable gravitational constant. Triggered by the development of the string and D-brane theories, some interesting new approaches to the compactification mechanism have recently been proposed [2, 3, 4, 5] and are applied to the hierarchy problem. Here we examine the Randall-Sundrum (RS) model which has some attractive features compared with the Kaluza-Klein compactification. The model is becoming a strong candidate that could solve the mass hierarchy problem. It looks, however, that the domain wall configuration is usually introduced "by hand" (not solving the field equation properly) and is often "approximated" by some distribution such as δ-function or θ-function[4, 5]. Such approximate approach sometimes hinders us from treating delicate (but important) procedures such as the boundary condition, the (infrared) regularization and the cosmological term. We present a solution of the field equation, which clarifies the compactification mechanism much more than the usual treatment. Especially the full-fledged treatment of the vacuum in terms of the 5D Higgs potential is an advantage. For the purpose of treating the model starting from the Lagrangian, we consider the model in a simplified case: One-wall model which was considered in [5]. An interesting *stable* (kink) solution is found for a *family* of vacua. The solution does not miss the key points of the original one. Some new features are 1) the (5D) cosmological constant has both the upper bound and the lower bound (15); 2) the wall-thickness parameter k (defined later) is regarded as the most important quantity to control the whole configuration properly and should be bounded both from below and from above (26); 3) As a plausible numerical choice, we propose $k \sim 10^4$GeV to explain quark, lepton and neutrino masses (Sec.5 and Sec.6).

The domain wall configuration, which is exploited in the RS model, has

been frequently discussed so far in the literature. Especially the relation between some anomalies is examined in [6]. The regularization of the chiral fermion problem on lattice was examined in [7, 8, 9, 10, 11]. The similarity to these works is shown by clarifying the parameters correspondence. The difference between them is only the interpretation of the extra axis; In the chiral fermion case it is regarded as a purely technical axis for the regularization, whereas, in the RS model, it is a physical axis whose size is too small to measure at present. The analysis using the RS model can be regarded as a geometrical approach to the chiral fermion problem.

At present there exists no sign of the extra dimension(s) experimentally. Hence one might wonder about the worth of the present line of research. We remind you, however, of the following important aspects behind.

1. The higher dimensional view to the present world has been taken , since Kaluza-Klein[12], by many physicists. The latest approaches are unified models based on the supergravity, the string and the D-brane. This is one basic standpoint to understand the nature from geometry. The RS model is one of such approaches and has many distinguished properties compared with the past ones.

2. Use of the extra dimension(s) is one possible ingredient, independent of the supersymmetry, which can make us go beyond the standard model.

3. Many aspects appearing in the RS model overlap with those in the recent 10-15 years development of the theoretical physics: the chiral fermion problem (the previous paragraph), anomaly problem, D-brane physics, AdS/CFT, etc..

4. As reported in ICHEP2000[13], some interesting phenomena await the experimental tests.

We introduce the present model in Sec.2. A solution is obtained in Sec.3, where it is essentially described by three parameters specifying the Higgs vacuum. In Sec.4, the asymptotic behaviors, in the dimensional reduction from 5D to 4D, are evaluated for the three parameters. In Sec.5, the orders of magnitude for the two quantities, the thickness parameter (k) and the 5D Planck mass (M), are examined from the information of the 4D Planck mass, the 4D cosmological constant and the present experimental

status of the Newton's law. In Sec.6, similarity between the present analysis and the the chiral fermion problem is pointed out. Using the 4D fermion (quarks,leptons) masses, we examine the value of the size of the extra dimension (r_c). The three parameters referred above are precisely determined in Sec.7, where two constraints coming from the boundary condition are solved for the parameters. We conclude and discuss in Sec.8.

2 Model set-up

We start with the 5D gravitational theory, where the metric is Lorenzian, with the 5D Higgs potential.

$$S[G_{AB},\Phi]=\int d^5X\sqrt{-G}(-\frac{1}{2}M^3\hat{R}-\frac{1}{2}G^{AB}\partial_A\Phi\partial_B\Phi-V(\Phi))\quad ,$$
$$V(\Phi)=\frac{\lambda}{4}(\Phi^2-{v_0}^2)^2+\Lambda\quad ,\qquad (1)$$

where $X^A(A=0,1,2,3,4)$ is the 5D coordinates and we also use the notation $(X^A)\equiv(x^\mu,y),\mu=0,1,2,3$. $X^4=y$ is the extra axis which is taken to be a space coordinate. The signature of the 5D metric G_{AB} is $(-++++)$. Φ is a 5D scalar field, $G=\det G_{AB}$, $\hat{R}$ is the 5D Riemannian scalar curvature. $M(>0)$ is the 5D Planck mass and is regarded as the *fundamental scale* of this dimensional reduction scenario. $V(\Phi)$ is the Higgs potential and serves for preparing the (classical) vacuum in 5D world. See Fig.1. The three parameters λ,v_0 and Λ in $V(\Phi)$ are called here *vacuum parameters*. $\lambda(>0)$ is a coupling, $v_0(>0)$ is the Higgs field vacuum expectation value, and Λ is the 5D cosmological constant. It is later shown that the sign of Λ must be negative for the proposed domain wall vacuum configuration. Following [4], we take the line element shown below.

$$ds^2=\mathrm{e}^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu+dy^2\quad ,\qquad (2)$$

where $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$. In this choice, the 4D Poincaré invariance is preserved. The "warp" factor $\mathrm{e}^{-2\sigma(y)}$ plays an important role throughout this paper. Note that, for the fixed y case ($dy=0$), the metric is the Weyl transformation of the flat (Minkowski) space $\eta_{\mu\nu}dx^\mu dx^\nu$ (See Sec.6).

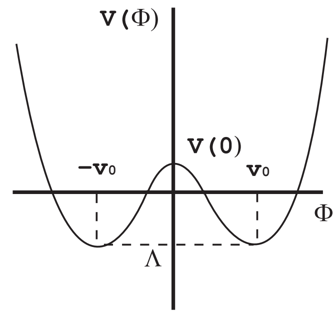

Fig.1 The Higgs Potential $V(\Phi)$, (1). Horizontal axes: Φ. From (15), $V(0)=\lambda {v_0}^4/4+\Lambda>0,\ V(v_0)=\Lambda<0$.

3 A solution

Let us solve the 5D Einstein equation.

$$M^3(\hat{R}_{MN}-\frac{1}{2}G_{MN}\hat{R})=-\partial_M\Phi\,\partial_N\Phi+G_{MN}(\frac{1}{2}G^{KL}\partial_K\Phi\,\partial_L\Phi+V(\Phi))\quad ,$$
$$\nabla^2\Phi=\frac{\delta V}{\delta\Phi}\quad . \qquad (3)$$

Following Callan and Harvey[6], we consider the case that Φ depends only on the extra coordinate y, $\Phi=\Phi(y)$. The above equations reduce to

$$-6M^3(\sigma')^2=-\frac{1}{2}(\Phi')^2+V\quad , \qquad (4)$$
$$3M^3\sigma''=(\Phi')^2\quad . \qquad (5)$$

We note that the "matter equation", the last one of (3), can also be obtained from $(M,N)=(4,4)$ component of the "gravitational equation", the first one of (3) which is given by (4). As the extra space (the fifth dimension), we take the real number space $\mathbf{R}=(-\infty,+\infty)$. This is a simplified version of the original RS-model[4] where $S^1/\mathbf{Z}_2$ is taken. We impose the following asymptotic behaviour for the (classical) vacuum of $\Phi(y)$.

$$\Phi(y)\to\pm v_0\quad ,\quad y\to\pm\infty \qquad (6)$$

This means $\Phi'\to 0$, and from (5), $\sigma''\to 0$. Integrating eq.(5), we obtain

$$3M^3\{\sigma'|_{y=+\infty}-\sigma'|_{y=-\infty}\}=\int_{-\infty}^{\infty}(\Phi')^2dy>0\quad . \qquad (7)$$

From this result, we are led to $\sigma' \to \pm\omega, \sigma \to \omega|y|$ as $y \to \pm\infty$, where $\omega(>0)$ is some constant to be determined soon. We can scale out M in (4) by rescaling all fields (Φ, σ), all vacuum parameters (λ, v_0, Λ) and the coordinate y with appropriate powers of M. $(\Phi = M^{3/2}\tilde{\Phi}, \sigma = \tilde{\sigma}, v_0 = M^{3/2}\tilde{v}_0, \lambda = M^{-1}\tilde{\lambda}, \Lambda = M^5\tilde{\Lambda}, X^A = M^{-1}\tilde{X}^A,)$ Therefore we may set $M = 1$ without ambiguity. (Only when it is necessary, we explicitly write down M-dependence.)

First we fix the parameter ω, by considering $y \to \pm\infty$ in (4), as

$$\omega = \sqrt{\frac{-\Lambda}{6}} M^{-\frac{3}{2}} \quad , \tag{8}$$

where we see the sign of Λ must be *negative*, that is, the 5D geometry must be *anti de Sitter* in the asymptotic regions.

Let us take the following form for $\sigma'(y)$ and $\Phi(y)$ as a solution.

$$\begin{aligned} \sigma'(y) &= k \sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!} \{\tanh(ky+l)\}^{2n+1} \quad , \\ \Phi(y) &= v_0 \sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!} \{\tanh(ky+l)\}^{2n+1} \quad , \end{aligned} \tag{9}$$

where c's and d's are coefficient-constants (with respect to y) to be determined. The free parameter l comes from the *translation invariance* of (4) and (5). A *new mass scale* $k(>0)$ is introduced here to make the quantity ky dimensionless. The physical meaning of $1/k$ is the "thickness" of the domain wall. The parameter k, with M and r_c(defined later), plays a central role in this dimensional reduction scenario. We call M, k and r_c *fundamental parameters*. The distortion of 5D space-time by the existence of the domain wall should be small so that the quantum effect of 5D gravity can be ignored and the present *classical* analysis is valid. This requires the condition[4]

$$k \ll M \quad . \tag{10}$$

The coefficient-constants c's and d's have the following constraints

$$\sqrt{\frac{-\Lambda}{6}} = k \sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!} \quad , \quad 1 = \sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!} \quad , \tag{11}$$

which are obtained by considering the asymptotic behaviours $y \to \pm\infty$ in (9). We will use these constraints in Sec.7.

We first obtain the recursion relations between the expansion coefficients, from the field equations (4) and (5). For $n \geq 2$, they are given by

$$\frac{c_{2n+1}}{(2n)!} - \frac{c_{2n-1}}{(2n-2)!} = \frac{{v_0}^2}{3}({D'}_n - 2{D'}_{n-1} + {D'}_{n-2}) \quad ,$$
$$-6C_{n-1} = -\frac{{v_0}^2}{2}({D'}_n - 2{D'}_{n-1} + {D'}_{n-2}) + \frac{\lambda}{4}\frac{{v_0}^4}{k^2}(E_{n-2} - 2D_{n-1}) \ , \qquad (12)$$

where

$$D_n = \sum_{m=0}^{n} \frac{d_{2n-2m+1}d_{2m+1}}{(2n-2m+1)!\,(2m+1)!} \quad , \quad {D'}_n = \sum_{m=0}^{n} \frac{d_{2n-2m+1}d_{2m+1}}{(2n-2m)!\,(2m)!} \ ,$$
$$C_n = \sum_{m=0}^{n} \frac{c_{2n-2m+1}c_{2m+1}}{(2n-2m+1)!\,(2m+1)!} \quad , \quad E_n = \sum_{m=0}^{n} D_{n-m}D_m \quad . \ (13)$$

The first few terms, $(c_1, d_1), (c_3, d_3)$, are explicitly given as

$$d_1 = \pm\frac{\sqrt{2}}{v_0 k}\sqrt{\Lambda + \frac{\lambda {v_0}^4}{4}} \quad , \quad c_1 = \frac{2}{3k^2}(\Lambda + \frac{\lambda {v_0}^4}{4}) \quad ,$$
$$\frac{d_3}{d_1} = 2 + \frac{1}{k^2}\{\frac{8}{3}(\Lambda + \frac{\lambda {v_0}^4}{4}) - \lambda {v_0}^2\} \ , \ \frac{c_3}{c_1} = 2 + \frac{1}{k^2}\{\frac{16}{3}(\Lambda + \frac{\lambda {v_0}^4}{4}) - 2\lambda {v_0}^2\} \ , (14)$$

where $\pm$ sign in d_1 reflects $\Phi \leftrightarrow -\Phi$ symmetry in (4) and (5). We take the positive one in the following. We can confirm that the above relations, (12) and (14), determine all c's and d's recursively in the order of increasing n. This is because eq.(12) is the coupled *linear* equation with respect to (c_{2n+1}, d_{2n+1}) when lower-order c's and d's are regarded as obtained quantities. (Note: $D'_n = \frac{2d_1}{(2n)!}d_{2n+1}$+lower-order terms, $D_n = \frac{2d_1}{(2n+1)!}d_{2n+1}$+lower-order terms, $C_n = \frac{2c_1}{(2n+1)!}c_{2n+1}$+lower-order terms, $E_n = \frac{8{d_1}^3}{(2n+1)!}d_{2n+1}$+lower-order terms.) All coefficients are solved and are described by the three dimensionless vacuum parameters:
$(\Lambda + \frac{\lambda {v_0}^4}{4})/k^2M^3,\ \lambda {v_0}^2/k^2,\ {v_0}^2/M^3$.

In order for this solution to make sense, as seen from the expression for d_1, the 5D cosmological term Λ should be bounded also from below, in addition to from above.

$$-\frac{\lambda {v_0}^4}{4} < \Lambda < 0 \quad . \qquad (15)$$

At this stage the two constraints (11) are not taken into account. These impose some relations between vacuum parameters which will be explained in Sec.7.

4 Vacuum parameters: M and k-dependence in the dimensional reduction

Let us examine the behaviour of the vacuum parameters (Λ, v_0, λ) near the 4D world: $k \to \infty$(the dimensional reduction). This should be taken consistently with (10). We will specify the above limit in the more well-defined way later. We take the following assumption which will be later checked using the final solution,

$$\frac{c_{2n+1}}{c_1} \to O(k^0) \times O(n^0) \quad , \quad \frac{d_{2n+1}}{d_1} \to O(k^0) \times O(n^0) \quad , \qquad \text{as} \quad k \to \infty \ , \ n \to \infty \quad , \tag{16}$$

where $O(k^0)$ and $O(n^0)$ are some constants of order k^0 and n^0. $O(n^0)$ behaviour for $n \to \infty$ is a sufficient condition for the convergence of the infinite series (9). Then the expressions (9) has the following asymptotic form, as $k \to \infty$.

$$\begin{aligned} \sigma'(y) &\to kc_1 \times \theta(ky) \times \text{const} = \frac{2}{3k}(\Lambda + \frac{\lambda {v_0}^4}{4})\,\theta(ky) \times \text{const} \ , \\ \Phi(y) &\to v_0 d_1 \times \theta(ky) \times \text{const} = \frac{\sqrt{2}}{k}\sqrt{\Lambda + \frac{\lambda {v_0}^4}{4}}\,\theta(ky) \times \text{const} \ , \end{aligned} \tag{17}$$

where $\theta(y)$ is the step function: $\theta(y) = 1$ for $y > 0, \theta(y) = -1$ for $y < 0$. (Note: $(\tanh ky)^{2n+1} \to \theta(ky),\ k \to \infty$.) Taking relations (6) and (8) into account, (17) means $\lambda {v_0}^4 \sim -\Lambda \sim k\sqrt{-\Lambda} \sim k^2 {v_0}^2$. These relations say

$$-\Lambda \sim M^3 k^2 \quad , \quad v_0 \sim M^{3/2} \quad , \quad \lambda \sim M^{-3} k^2 \quad \text{as} \quad k \to \infty \quad . \tag{18}$$

These are *leading* behaviour of the vacuum parameters in the dimensional reduction. The first one above is given in the original [4]. The more precise forms of (18) will be obtained, in Sec.7, using the constraints (11).

5 Parameter fitting

In order to express some physical scales in terms of the fundamental parameters M, k and r_c(to be introduced soon), we consider the case that the 4D geometry is slightly fluctuating around the Minkowski (flat) space.

$$ds^2 = \mathrm{e}^{-2\sigma(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2 \quad , \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad , \quad h_{\mu\nu} \sim O(\frac{1}{k}) \quad . \tag{19}$$

The leading order $O(k^0)$ results of the previous section remain valid.

5.1 The Planck mass

The gravitational part of 5D action (1) reduces to 4D action as

$$\int d^5X\sqrt{-G}M^3\hat{R} \sim M^3 \int_{-r_c}^{r_c} dy \mathrm{e}^{-2\sigma(y)} \int d^4x\sqrt{-g}R + \cdots \quad , \tag{20}$$

where the *infrared regularization* parameter r_c is introduced. r_c specifies the *length* of the extra axis. Using the asymptotic forms, $\sigma(y) \sim \omega|y|$ as $y \to \pm\infty$ and $\omega = \sqrt{\frac{-\Lambda}{6}} M^{-\frac{3}{2}} \sim k$ as $k \to \infty$, we can evaluate the order of M_{pl} as

$${M_{pl}}^2 \sim M^3 \int_{-r_c}^{r_c} dy\, \mathrm{e}^{-2\omega|y|} = \frac{M^3}{\omega}(1 - \mathrm{e}^{-2\omega r_c}) \sim \frac{M^3}{k} \quad , \tag{21}$$

where we have used the *4D reduction condition*:

$$kr_c \gg 1 \quad . \tag{22}$$

The result (21) is again same as in [4]. The above condition should be interpreted as the precise (well regularized) definition of $k \to \infty$ used so far. We note r_c dependence in (21) is negligible for $kr_c \gg 1$. This behaviour shows the distinguished contrast with the Kaluza-Klein reduction (${M_{pl}}^2 \sim M^3 r_c$) as stressed in [4].

5.2 The cosmological term

The cosmological part of (1) reduces to 4D action as

$$\int d^5X\sqrt{-G}\Lambda \sim \Lambda \int_{-r_c}^{r_c} dy\, \mathrm{e}^{-4\sigma(y)} \int d^4x\sqrt{-g} \equiv \Lambda_{4d} \int d^4x\sqrt{-g} \quad ,$$

$$\Lambda_{4d} \sim \frac{\Lambda}{2\omega}(1 - \mathrm{e}^{-4\omega r_c}) \sim -M^3 k < 0 \quad , \quad kr_c \gg 1 \quad . \tag{23}$$

Λ_{4d} is the cosmological term in the 4D space-time. It does not, like M_{pl}, depend on r_c strongly. The result says the 4D space-time should also be *anti de Sitter*.

5.3 Numerical fitting

Let us examine what orders of values should we take for the fundamental parameters M and k. (r_c is later fixed by the information of the 4D fermion masses.) Using the value $M_{pl} \sim 10^{19}$GeV , the "rescaled" cosmological parameter $\tilde{\Lambda}_{4d} \equiv \Lambda_{4d}/{M_{pl}}^2$ [14] has the relation:

$$\sqrt{-\tilde{\Lambda}_{4d}} \sim k \sim M^3 \times 10^{-38} \text{ GeV} \quad , \tag{24}$$

where the relations (21) and (23) are used. The unit of M is GeV and this mass unit is taken in the following. The observed value of $\tilde{\Lambda}_{4d}$ is not definite, even for its sign.[15] If we take into account the quantum effect, the value of $\tilde{\Lambda}_{4d}$ could run along the renormalization[16]. Furthermore if we consider the parameter $\tilde{\Lambda}_{4d}$ represents some "effective" value averaging over all matter fields, the value, no doubt, changes during the evolution of the universe. (Note the model (1) has no (ordinary) matter fields.) Therefore, instead of specifying $\tilde{\Lambda}_{4d}$, it is useful to consider various possible cases of $\tilde{\Lambda}_{4d} \sim -k^2$. Some typical cases are 1) $(k = 10^{-41}, M = 0.1)$, 2) $(k = 10^{-13}, M = 10^8)$ 3) $(k = 10, M = 10^{13})$ 4) $(k = 10^4, M = 10^{14})$ and 5) $(k = 10^{19}, M = 10^{19})$. Case 1) gives the most plausible present value of the cosmological constant. The wall thickness $1/k = 10^{41}$[GeV^{-1}] , however, is the radius of the present universe. This implies the extra dimensional effect appears at the cosmological scale, which should be abandoned. Case 2) gives $1/k = 10^{13}$ GeV^{-1} $\sim$ 1mm which is the minimum length at which the Newton's law is checked[2]. Usually k should be larger than this value so that we keep the observed Newton's law. 5) is an extreme case $M = M_{pl}$. The fundamental scale M is given by the Planck mass. In this case, $r_c \gg 1/k = 1/M_{pl}$ is acceptable, while $\sqrt{-\tilde{\Lambda}_{4d}} \sim M_{pl}$ is completely inconsistent with the experiment and requires explanation. Most crucially the condition (10) breaks down. Cases 3) and 4) are some intermediate cases which are acceptable except for the cosmological constant. They will be used in Sec.8. At present any choice of (k, M) looks to have some trouble if we take into account the cosmological constant. We consider the observed cosmological constant (10^{-41}GeV) should

be explained by some unknown mechanism. (No successful explanation of the small cosmological constant exists [18]. Ordinarily (without fine-tuning) the quantum-loop correction leads to the case 5)[19]. Compared with case 5), the cases 3) and 4) should be regarded as "much improved" cases in this respect.)

6 Domain wall in the chiral fermion problem

We point out the mechanism presented here has a strong similarity to that in the chiral fermion determinant. The *interpretation* of the extra axis only is the difference. The axis is regarded as a real (but hardly measurable) axis here, whereas it is a regularization axis in the chiral problem. The parameter correspondence is

$$
\begin{array}{rcl}
\text{Randal-Sundrum} & & \text{Chiral Fermion[11, 20, 21]} \\
k:\ (\text{thickness of the wall})^{-1} & \leftrightarrow & M_F:\ \text{1+4 dim fermion mass or} \\
 & & \ (\text{thickness of the wall})^{-1} \\
M:\ \text{fundamental scale} & \leftrightarrow & 1/t:\ \text{temperature or} \\
 & & 1/a:\ (\text{lattice spacing})^{-1} \\
r_c:\ \text{Infrared reg.} & \leftrightarrow & 1/|k^\mu|:\ (\text{4D fermion mom.})^{-1}\text{or} \\
= \text{size of the extra axis} & & 1/m_q:\ (\text{quark mass})^{-1}
\end{array}
\qquad (25)
$$

The condition on k in the RS model, from (10) and (22), is given as

$$\frac{1}{r_c} \ll k \ll M \quad . \qquad (26)$$

The corresponding one of the chiral fermion is given by[20, 21]

$$|k^\mu| \ll M_F \ll \frac{1}{t} \quad . \qquad (27)$$

Both conditions guarantee the mechanism effectively works.

It is known, in the lattice chiral fermion, the choice of the parameter M_F is so important to produce a good numerical output, say, the pion mass([22] where M_F is denoted as M_5 and is called "domain wall height"). Only

for well-chosen value of M_F, the chiral properties are controlled. In this analogy the thickness parameter k, in the RS model, is considered to be a key quantity for controlling the whole configuration and for fitting with the real world quatities such as fermion masses.

The line element of (2) or (19) for a fixed y is the Weyl scaling $g_{\mu\nu}(x) \rightarrow \mathrm{e}^{-2\sigma(y)} g_{\mu\nu}(x)$ of the 4D world: $(ds^2)_{4D} = g_{\mu\nu}(x)dx^\mu dx^\nu$. $\sigma(y)$ is related to the 4D dynamics through the 5D geometrical setting. The extra dimension y plays the role of the *scaling* parameter. On the other hand, in the chiral problem, the extra axis can be regarded as the Schwinger's proper time (inverse temperature) t[23] through the relation [20, 21]:

$$(\frac{\partial}{\partial t} + \hat{D})G(x,y;t) = 0 \quad , \quad G(x,y;t) = < x|\mathrm{e}^{-t\hat{D}}|y > \quad , \tag{28}$$

where $\hat{D}$ is the general 4D operator and $G(x,y;t)$ is the density matrix. Formally it says $\frac{\partial G}{\partial t} \cdot G^{-1} = \frac{\partial}{\partial t} \ln G = -\hat{D}$. This shows the *scaling* property of $\ln G$ along the coordinate t. These similar roles of y and t strongly indicate the both mechanisms are essentially the same.

In the view of [20, 21], the "direction" of the system evolvement of the present model is given by the sign change of the 5D Higgs field around the origin $y = 0$.

As in the Callan and Harvey's paper[6], we can have the 4D *massless chiral* fermion bound to the wall by introducing 5D *Dirac* fermion ψ into (1).

$$S[G_{AB}, \Phi] + \int d^5X \sqrt{-G}(\bar{\psi}\nabla\!\!\!\!/\,\psi + g\Phi\bar{\psi}\psi) \quad . \tag{29}$$

If we *regulate* the extra axis by the finite range $-r_c \leq y \leq r_c$, the 4D fermion is expected to have a small mass $m_f \sim k\mathrm{e}^{-kr_c}$ (This is known for the two-walls case in [9, 11]). If we take the case 3) in Subsec.5.3 ($k = 10, M = 10^{13}$) and regard the 4D fermion as a neutrino ($m_\nu \sim 10^{-11} - 10^{-9}$GeV), we obtain $r_c = 2.76 - 2.30\mathrm{GeV}^{-1}$. If we take case 4) ($k = 10^4, M = 10^{14}$), we obtain $r_c = (3.45 - 2.99) \times 10^{-3}\mathrm{GeV}^{-1}$. When the quarks or other leptons ($m_q, m_l \sim 10^{-3} - 10^2$GeV) are taken as the 4D fermion, and take the case 4) in Subsec.5.3, we obtain $r_c = (1.61 - 0.461) \times 10^{-3}\mathrm{GeV}^{-1}$. It is quite a fascinating idea to *identify the chiral fermion zero mode bound to the wall with the neutrinos, quarks or other leptons.*

7 Precise form of vacuum parameters

As shown in (18), an interesting aspect of the present solution is that some family of vacua is selected as the consistent (classical) configuration. Let us determine the precise form of (18) using the two constraints (11). In terms of new parameters $\Omega \equiv \Lambda + \frac{\lambda}{4}{v_0}^4 (0 < \Omega < \frac{\tau}{4}{v_0}^2), \tau \equiv \lambda {v_0}^2$, instead of Λ and λ, the precise forms are obtained by the $\frac{1}{k^2}$-expansion for the case $kr_c \gg 1$ as

$$
\begin{aligned}
\Omega = M^3k^2(\alpha_0 + \frac{\alpha_1}{(kr_c)^2} + \cdots) &= M^3k^2\sum_{n=0}^{\infty}\frac{\alpha_n}{(kr_c)^{2n}} \quad , \\
\tau = k^2(\gamma_0 + \frac{\gamma_1}{(kr_c)^2} + \cdots) &= k^2\sum_{n=0}^{\infty}\frac{\gamma_n}{(kr_c)^{2n}} \quad , \\
v_0 = M^{3/2}(\beta_0 + \frac{\beta_1}{(kr_c)^2} + \cdots) &= M^{3/2}\sum_{n=0}^{\infty}\frac{\beta_n}{(kr_c)^{2n}} \quad ,
\end{aligned}
\tag{30}
$$

where α's,γ's and β's are some numerical (real) numbers to be consistently chosen using (11). If we assume the relation (16), the infinite series of (11) can be safely truncated at the first few terms. In order to demonstrate how the vacuum parameters are fixed, we take into account up to $n = 2$ in (11) and up to $O(1/(kr_c)^{2\times 2})$ in (30). For general M, k, r_c except the condition $kr_c \gg 1$, the coefficients are determined as

$$
\begin{aligned}
&\text{Vacuum 1: } (\alpha_0, \alpha_1, \alpha_2) \equiv (1, 0, 0) \text{ input} \\
&\qquad (\beta_0, \beta_1, \beta_2; \gamma_0, \gamma_1, \gamma_2) = (1.6, 0, 0; 4.2, 0, 0) \quad , \\
&\text{Vacuum 2: } (\alpha_0, \alpha_1, \alpha_2) \equiv (1, 1, 0) \text{ input} \\
&\qquad (\beta_0, \beta_1, \beta_2; \gamma_0, \gamma_1, \gamma_2) = (1.6, 1.1, -0.67; 4.2, 1.8, 1.7) \quad , \\
&\text{Vacuum 3: } (\alpha_0, \alpha_1, \alpha_2) \equiv (1, 1, 1) \text{ input} \\
&\qquad (\beta_0, \beta_1, \beta_2; \gamma_0, \gamma_1, \gamma_2) = (1.6, 1.1, 0.45; 4.2, 1.8, 3.5) \quad .
\end{aligned}
\tag{31}
$$

We notice our solution has one *free parameter* for each n-th set $(\alpha_n, \beta_n, \gamma_n)$. This is because the number of constraints for c's and d's is two (11), whereas that of quantities to be determined is three (30). Using this freedom we can adjust one of the three vacuum parameters in the way the observed physical values are explained. In (31), we take α's as the input. Taking the value $kr_c = 10$, Vac.3 has the vacuum expectation value $v_0M^{-3/2} = 1.6$, the cosmological

constant $\Lambda k^{-2}M^{-3} = -1.7$ and the coupling $\lambda k^{-2}M^3 = 1.6$. Other vacua have almost the same values because the first order term dominate in (30) for the thin wall case $kr_c \gg 1$. We notice the dimensionless vacuum parameters, $v_0 M^{-3/2}$, $\Lambda k^{-2}M^{-3}$ and $\lambda k^{-2}M^3$, are specified only by the value of kr_c and the input data, say, α's. If we specify k and M, as considered in Subsec 5.3, the values v_0, Λ and λ are obtained. Any higher-order, in principle, can be obtained by the $\frac{1}{k^2}$-expansion.

For the Vac.3, we plot $\Phi(y)$ and $\Phi'(y)$ for two cases $kr_c = 10$ and $kr_c = 20$ in Fig.2.

8 Discussion and conclusion

The assumption used in the present explanation is (16) only. The first some values in the series of n: $(c_3/c_1, d_3/d_1) = (-1.05, 0.48), (c_5/c_1, d_5/d_1) = (-11.9, -0.90)$ for Vac.3 with $kr_c = 10$ indicate its validity. Another evidence of the (strong) convergence is the fact that the normalization in Fig.2 is quite correctly reproduced.

If we take the boundary condition: $\Phi(y) \rightarrow \mp v_0 \ , \ y \rightarrow \pm\infty$, instead of (6), the opposite *chirality* solution is obtained. Both of the pair, + and − chiralities, are indispensable when the "vector-like" or non-chiral theory, such as QCD, is taken into account.

One of the fundamental parameters, r_c, is introduced in Sec.5 and 6 as the infrared regularization. This is quite natural in the standpoint of the discretized approach such as lattice. The treatment, however, should be regarded as an "effective" approach or a "temporary" stage of the unknown right treatment. The scale r_c should be introduced naturally in the continuum approach. If we can generalize the present analysis to the case of the S^1/Z_2 extra space (two-walls case), r_c is interpreted as the distance between the two walls [4]. Another interesting possibility is that the scale r_c could be given by some (at present) unknown mechanism in the space-time manifold such as the non-commutative geometry[24]. It also looks that the AdS/CFT view[25] of the present model could give a clue to the problem.

An important task to establish the RS scenario is to introduce the standard electro-weak model (chiral), QCD (non-chiral) and SUSY theories into this scheme. Recently the bulk standard model has been examined by [26]. In [27] a supersymmetric extension is examined. The RS model has given us

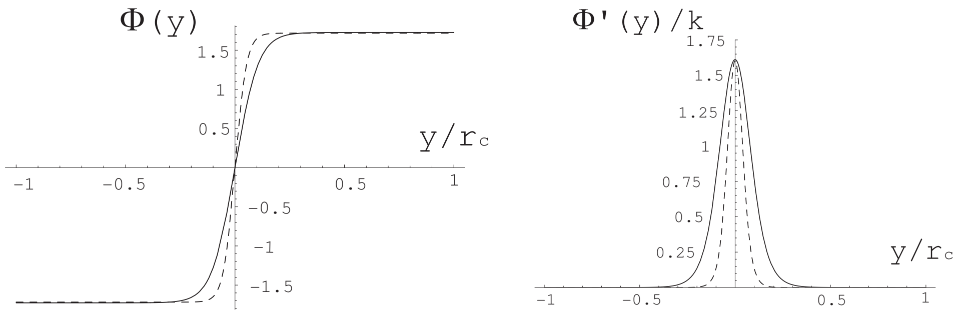

Fig.2 Vertical axes: 5D Higgs field ($\Phi(y)$) and its derivative ($\Phi'(y)/k$); Horizontal axes: y/r_c; Vacuum 3 of (31); Solid line: $kr_c = 10$, Dotted line: $kr_c = 20$.

richer possibilities for the mass hierarchy problem than before. It is hoped that the future experiments can select them.

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